# SSB – Weaver method

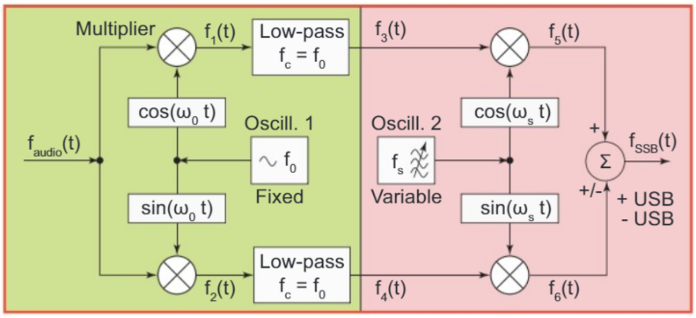


FIGURE 1: The block diagram of a Weaver SSB modulator.

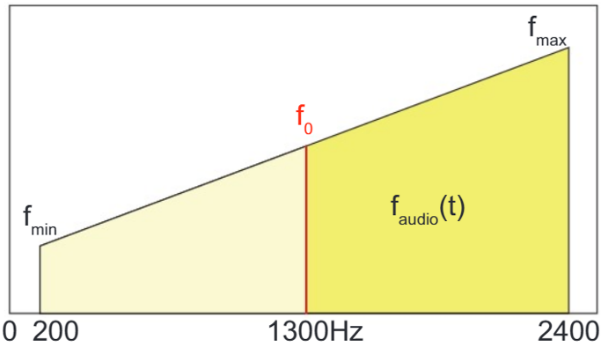


FIGURE 2: The frequency spectrum of a typical SSB audio signal.

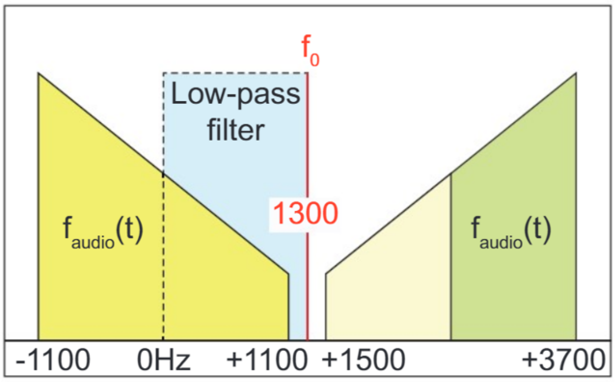


FIGURE 3: Frequency spectrum of multplication products $f_1(t)$ and $f_2(t)$.

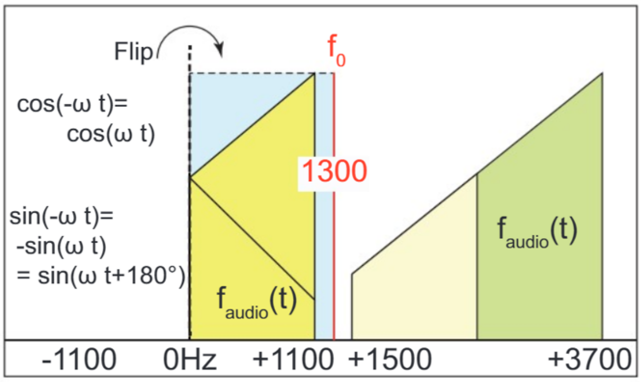


FIGURE 4: An alternative representation of spectrum $f_1(t)$ and $f_2(t)$.

**Anyone engaged with SDR solutions will sooner or later gain an interest in SSB modulation.**

In 1956, D.K.Weaver published a new method to generate single sideband signals (SSB) with suppressed carrier and demodulation. The hardware is very simple consisting of IQ-modulators and lowpass filters. It is the favorite method for digital implementations. It's called the 3rd method – following the filter- and phase-method. The first two methods require much more hardware but are less difficult to understand by electronic hobbyists. This article tries to explain the Weaver method avoiding too much higher maths.

## Modulation

**Figure 1** shows the block diagram of a Weaver SSB modulator. A band limited audio signal $f_{audio}(t)$ is fed to two multiplipliers. The additional injected oscillator signals $f_0(t)$ are shifted by 90° (sin, cos). The output signals of the multipliers $f_1(t)$ and $f_2(t)$ pass through identical lowpass filters with cutoff frequency $f_c = f_0$ and are inserted into another two multipliers fed with sin and cos versions of an auxiliary frequency $f_s$ as well. Adding output signals $f_5(t)$ and $f_6(t)$ results in the higher signal sideband (USB) and subtracting in the lower sideband signal (LSB).

Next, we focus on the modulation products of the multipliers. In **Table 1** you will find some important trigonometrical identities we use. In order to make things clearer, amplitudes and phase values are dropped. That means we set all levels to 1 (100%) and all phase angles to 0°.

**Figure 2** shows the frequency spectrum of a typical SSB audio signal. $f_0$ stands for the centre frequency of the audio range with $f_0 = (f_{min}+f_{max})/2$.

Applying formulas [TR1] and [TR3], from Table 1, to the multiplier input signals results in output signals $f_1(t)$ and $f_2(t)$. Both signals have the same amplitudes but different phases.

$$
\begin{aligned}
f_1(t) &= f_{audio}(t)\cos(2\pi f_0 t) \qquad |\ \omega = 2\pi f \\
&= \cos(\omega_{audio}t)\cos(\omega_0 t) \\
&= \left[\cos\left((\omega_{audio}-\omega_0)t\right)+\cos\left((\omega_{audio}+\omega_0)t\right)\right]/2 \quad [1]
\end{aligned}
$$

$$
\begin{aligned}
f_2(t) &= f_{audio}(t)\sin(2\pi f_0 t) \\
&= \cos(\omega_{audio}t)\sin(\omega_0 t) \\
&= \left[-\sin\left((\omega_{audio}-\omega_0)t\right)+\sin\left((\omega_{audio}+\omega_0)t\right)\right]/2 \quad [2]
\end{aligned}
$$

Entering an audio centre frequency $f_0$ of 1300Hz and an audio frequency range from 200Hz to 2400Hz in formulas [1] and [2] results in positive and negative frequencies as well (**Figure 3**).

Anyone not familiar with higher maths will have problems picturing negative frequencies. Formulas [TR4] in Table 1 gives us some help. A cosine signal with a negative argument is just the same as one with a positive argument. A sine signal with a negative argument is equal to one with a positive argument, but shifted by 180° in phase (negative sign).

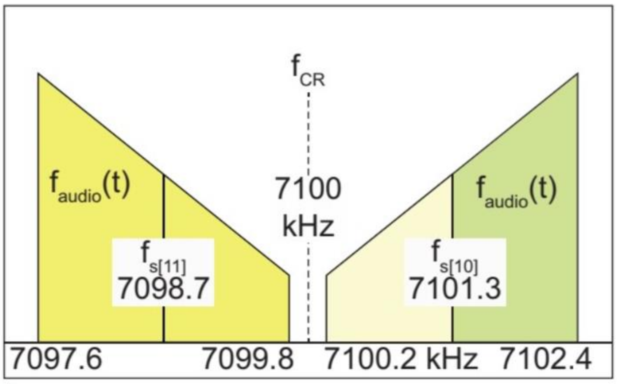


FIGURE 5: SSB spectrum with suppressed carrier frequency of 7100kHz.

That means all negative frequencies are folded over on the 0Hz-line to the positive range as shown in **Figure 4**.

Modulation products $f_1(t)$ and $f_2(t)$ run through identical low-pass filter with a cutoff frequency $f_c = f_0 = 1300$Hz. All arguments of the spectrum $> f_c$ will be rejected as shown by equations [3] and [4].

$$f_3(t) = \cos\left(\left(\omega_{audio} - \omega_0\right)t\right) \quad [3]$$

$$f_4(t) = -\sin\left(\left(\omega_{audio} - \omega_0\right)t\right) \quad [4]$$

$f_3(t)$ and $f_4(t)$ are the input signals to another pair of multipliers together with an auxiliary oscillator frequency of $f_S$. Output mixer products are shown in the mathematical equations [5] and [6] applying trigonometrical identities [TR1] and [TR2].

$$f_5(t) = f_s(t)f_3(t) = \cos(\omega_s t)\cos(\omega_3 t)$$
$$= \left[\cos\left(\left(\omega_s - \omega_{audio} + \omega_0\right)t\right) + \cos\left(\left(\omega_s + \omega_{audio} - \omega_0\right)t\right)\right]/2 \quad [5]$$

$$f_6(t) = f_s(t)f_4(t) = \sin(\omega_s t)\sin(-\omega_4 t)$$
$$= \left[-\cos\left(\left(\omega_s - \omega_{audio} + \omega_0\right)t\right) + \cos\left(\left(\omega_s + \omega_{audio} - \omega_0\right)t\right)\right]/2 \quad [6]$$

Addition of equations [5] and [6] gives the USB signal [7], subtraction the LSB signal [8].

$$f_5(t) = \left[\cos\left(\left(\omega_s - \omega_{audio} + \omega_0\right)t\right) + \cos\left(\left(\omega_s + \omega_{audio} - \omega_0\right)t\right)\right]/2 \quad [5]$$

$$f_6(t) = \left[-\cos\left(\left(\omega_s - \omega_{audio} + \omega_0\right)t\right) + \cos\left(\left(\omega_s + \omega_{audio} - \omega_0\right)t\right)\right]/2 \quad [6]$$

Addition: $$f_{USB}(t) = f_5(t) + f_6(t) = \cos\left(\left(\omega_s + \omega_{audio} - \omega_0\right)t\right) \quad [7]$$

Subtraction: $$f_{LSB}(t) = f_5(t) - f_6(t) = \cos\left(\left(\omega_s - \omega_{audio} + \omega_0\right)t\right) \quad [8]$$

Equation [9] represents the mathematical notation of a SSB signal whereby the suppressed carrier frequency is termed $f_{CR}$ respectively $\omega_{CR}$.The plus sign stands for the upper sideband USB, the minus sign for the lower sideband LSB.

$$f_{SSB}(t) = \cos\left(\left(\omega_{CR} \pm \omega_{audio}\right)t\right) \quad [9]$$

The arguments of the cosine function in equations [7] and [8], $\omega_S$ and $\omega_0$, are constant when transmitting in SSB mode, just the audio frequencies in $\omega_{audio}$ change. In sum and difference they represent the suppressed carrier frequency $\omega_{CR}$ [10], [11], summarised in equation [12].

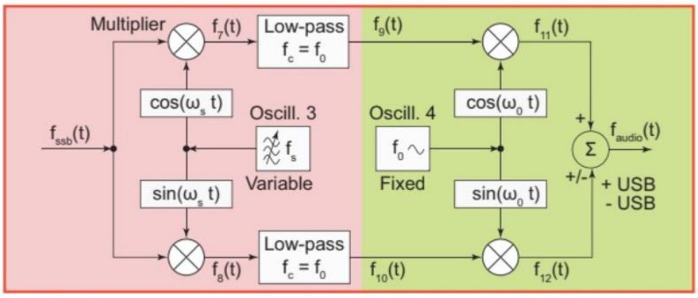


FIGURE 6: Block diagram of a Weaver SSB demodulator.

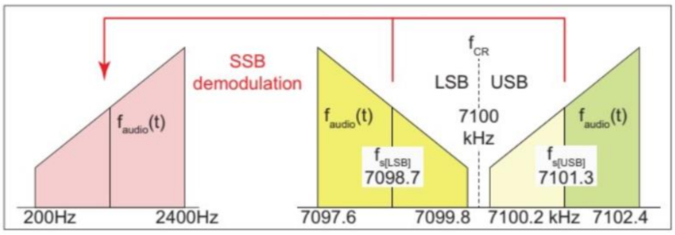


FIGURE 7: Demodulation spectrum.

$$f_{USB}(t) = f_5(t) + f_6(t) = \cos\left(\left(\omega_{CR} + \omega_{audio}\right)t\right) \quad | \quad \omega_{CR} = \omega_s - \omega_0 \quad [10]$$

$$f_{LSB}(t) = f_5(t) - f_6(t) = \cos\left(\left(\omega_{CR} - \omega_{audio}\right)t\right) \quad | \quad \omega_{CR} = \omega_s + \omega_0 \quad [11]$$

$$f_{SSB}(t) = \cos(\omega_{SSB}t) \quad | \quad \omega_{SSB} = \omega_{CR} \pm \omega_{audio} \quad [12]$$

Changing sidebands during an SSB transmission requires swapping addition and subtraction as well as adjusting frequency $f_S$.

For example: Transmitting on 7100kHz with an audio centre frequency of 1300Hz, the auxiliary frequency $f_S$ must be 7098.7kHz for LSB and 7101.3kHz for USB. When using digital signal generation this represents no problem, see **Figure 5**.

## Demodulation

Interchanging the frequencies of oscillators 1 and 2 turns the Weaver modulator (Figure 1) to a Weaver demodulator (**Figure 6**). The SSB signal at the input of the demodulator is multipied by the quadrature oscillator pair $\cos(\omega_S)$ and $\sin(\omega_S)$.

We get the multiplier output signals $f_7(t)$ and $f_8(t)$ by application of the trigonometric identities [TR1 ], [TR3] and [TR4] in Table 1.

$$f_7(t) = f_{SSB}(t)\cos(\omega_s t) = \cos(\omega_{SSB}t)\cos(\omega_s t) \quad | \quad \omega = 2\pi f$$
$$= \left[\cos\left(\left(\omega_{SSB} - \omega_s\right)t\right) + \cos\left(\left(\omega_{SSB} + \omega_s\right)t\right)\right]/2 \quad [13]$$

$$f_8(t) = f_{SSB}(t)\sin(\omega_s t) = \cos(\omega_{SSB}t)\sin(\omega_s t)$$
$$= \left[-\sin\left(\left(\omega_{SSB} - \omega_s\right)t\right) + \sin\left(\left(\omega_{SSB} + \omega_s\right)t\right)\right]/2 \quad [14]$$

We assume the lower frequency components

$$\cos\left(\left(\omega_s - \omega_{SSB}\right)t\right)$$

in [13] and

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$$\sin\left(\left(\omega_s - \omega_{SSB}\right)t\right)$$

in [14] pass the low-pass filters and the higher components

$$\cos\left(\left(\omega_s + \omega_{SSB}\right)t\right)$$

and

$$\sin\left(\left(\omega_s + \omega_{SSB}\right)t\right)$$

are rejected. Filtering this way yields to signals $f_9(t)$ and $f_{10}(t)$.

$$f_9(t) = \cos\left(\left(\omega_{SSB} - \omega_s\right)t\right) \quad [15]$$

$$f_{10}(t) = -\sin\left(\left(\omega_{SSB} - \omega_s\right)t\right) \quad [16]$$

Signals [15] and [16] multiplied by the quadrature oscillator pair $\cos(\omega_0)$ and $\sin(\omega_0)$ output signals $f_{11}(t)$ and $f_{12}(t)$.

$$f_{11}(t) = f_9(t)\cos\left(\omega_0 t\right) = \cos\left(\left(\omega_{SSB} - \omega_s\right)t\right)\cos\left(\omega_0 t\right) \quad | \quad \omega = 2\pi f$$
$$= \left[\cos\left(\left(\omega_{SSB} - \omega_s - \omega_0\right)t\right) + \cos\left(\left(\omega_{SSB} - \omega_s + \omega_0\right)t\right)\right]/2 \quad [17]$$

$$f_{12}(t) = f_{10}(t)\sin\left(\omega_0 t\right) = -\sin\left(\left(\omega_{SSB} - \omega_s\right)t\right)\sin\left(\omega_0 t\right)$$
$$= \left[-\cos\left(\left(\omega_{SSB} - \omega_s + \omega_0\right)t\right) + \cos\left(\left(\omega_{SSB} - \omega_s - \omega_0\right)t\right)\right]/2 \quad [18]$$

The addition of equations [17] and [18] generates equation [19], subtraction equation [20].

$$f_{11}(t) = \left[\cos\left(\left(\omega_{SSB} - \omega_s - \omega_0\right)t\right) + \cos\left(\left(\omega_{SSB} - \omega_s + \omega_0\right)t\right)\right]/2 \quad [17]$$
$$f_{12}(t) = \left[-\cos\left(\left(\omega_{SSB} - \omega_s + \omega_0\right)t\right) + \cos\left(\left(\omega_{SSB} - \omega_s - \omega_0\right)t\right)\right]/2 \quad [18]$$

$$\text{Addition:} \quad f_{audio}(t) = f_{11}(t) + f_{12}(t) = \cos\left(\left(\omega_{SSB} - \omega_s + \omega_0\right)t\right) \quad [19]$$
$$\text{Subtraction:} \quad f_{audio}(t) = f_{11}(t) - f_{12}(t) = \cos\left(\left(\omega_{SSB} - \omega_s - \omega_0\right)t\right) \quad [20]$$

**TABLE 1: Applied Trigonometrical Identities.**

$$\cos(x)\cos(y) = \left[\cos(x-y) + \cos(x+y)\right]/2 \quad [TR1]$$
$$\sin(x)\sin(y) = \left[\cos(x-y) - \cos(x+y)\right]/2 \quad [TR2]$$
$$\cos(x)\sin(y) = \left[\sin(x+y) - \sin(x-y)\right]/2 \quad [TR3]$$
$$\cos(-x) = \cos(x) \quad | \quad \sin(-x) = -\sin(x) \quad [TR4]$$

Equations [21] and [22] combine definitions made in equation [10] to [12].

$$\omega_{SSB(USB)} = \omega_s - \omega_0 + \omega_{audio} \quad [21]$$
$$\omega_{SSB(LSB)} = \omega_s + \omega_0 - \omega_{audio} \quad [22]$$

Substitutions [21] and [22] are inserted in equations [19] and [20] to eliminate variables $\omega_s$ and $\omega_0$. Left over is variable $\omega_{audio}$ [23], [24].

$$f_{audio(USB)}(t) = \cos\left(\omega_{audio} t\right) \quad [23]$$
$$f_{audio(LSB)}(t) = \cos\left(-\omega_{audio} t\right) \quad [24]$$

The negative sign in [24] does not matter as defined in Table 1 [TR4]. The effect is the same as swapping red and black poles on a speaker box: none! **Figure 7** illustrates the mathematical derivation using frequencies in the 40m band.

For implementation of the Weaver method in technical devices there are three possibilities:

- analogue
- digital
- hybrid

For analogue implementations, all multipiers are replaced by balanced mixers and analogue oscillators. Low-pass filters may be realised with RC or LC components.

Digital solutions use digital signal processing (DSP) and components like AD/DA-converters, FPGAs and microprocessors controlled by software.

Hybrid inplementions realise the method by using digital components for the low frequency part (green) and analog components for the higher frequency part (red) in block diagrams.

I hope to have contributed to a better understanding of the Weaver method. If any questions remain you can contact me by email.

**TABLE 2: Illustrating some types of amplitude modulation.**

$$f_{AM}(t) = A_0\cos\left(\omega_0 t + \phi_0\right)$$
$$+A_0 m\left[\cos\left(\omega_0 t + \omega_a t + \phi_0 + \phi_a\right)\right]/2 + A_0 m\left[\cos\left(\omega_0 t - \omega_a t + \phi_0 - \phi_a\right)\right]/2$$
$$f_{DSB}(t) = A_0 A_a\cos\left(\omega_0 t + \omega_a t + \phi_0 + \phi_a\right) + A_0 A_a\cos\left(\omega_0 t - \omega_a t + \phi_0 - \phi_a\right)$$
$$f_{SSB}(t) = A_0 A_a\cos\left(\omega_0 t \pm \omega_a t + \phi_0 + \phi_a\right) \quad | \quad \text{LSB (-), USB (+)}$$

$\omega_0$, $\omega_a$ are carrier, audio frequencies respectively

$A_0$, $A_a$ are carrier, audio amplitudes respectively

m is the modulation index

$\phi_0$, $\phi_a$ are carrier, audio phases respectively